Description of Multi-Particle Entanglement through Polynomial Invariants

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Entanglement is one of the major resources for both quantum communication and quantum computation. As for any resource, it is natural to ask how entanglement can be described and quantified. Several different measures of entanglement have been proposed. Any such quantification of entanglement implies a total ordering on the set of states, but the entanglement of multi-particle states could be incomparable. Thus we do not aim at a *quantification*, but at a *description* of entanglement which at least allows to decide whether two states are equally entangled. More precisely, we consider two states to be *locally equivalent* if they are related by local unitary transforms (LUT). In order to describe the orbits of states under LUT, we use means of invariant theory.

In the talk, we will present methods to compute polynomial local invariants for pure and for mixed quantum states with respect to LUT [1]. The structure of the ring of all polynomial invariants is partially reflected by the *Molien series*, a formal power series encoding the vector space dimension of homogeneous invariants of fixed degree. One obstacle that had to be circumvented was the fact that invariants involving complex conjugation—such as the norm of a vector—are in general not polynomial invariants. Therefore, additional—"complex conjugated"—variables have to be introduced. This yields a Molien-type series in two variables. We will discuss how to compute the Molien and Molien-type series for tensor products of both unitary groups and the special unitary groups. Examples for up to four qubits are given. For pure states of three qubits the structure of the invariant rings for both the unitary and special unitary groups are analyzed. For mixed states, the simplest case of two qubits is considered. Finally, recent intermediate results for pure states of four qubits and of three qutrits will be presented, followed by a discussion of limitations of the method and open problems.

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[1] Markus Grassl, Martin Rötteler, and Thomas Beth. Computing Local Invariants of Quantum–Bit Systems. *Physical Review A*, 58(3):1833–1839, September 1998. See also LANL preprint quant–ph/9712040.