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Invited Session: Quantum computation

Methods of Quantum Error Correction

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Arbeitsgruppe *Quantum Computing*

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Outline

- Statement of the problem
- Quantum channels
- Quantum error-correcting codes (QECC)
- Communication via Teleportation
- Entanglement purification & entanglement swapping
- Quantum repeaters
- Related topics

Basic Problem

storage, transmission, stabilization of quantum information, quantum states

quantum state $\Longrightarrow$ quantum channel $\Longrightarrow$ erroneous quantum state

$\Uparrow$

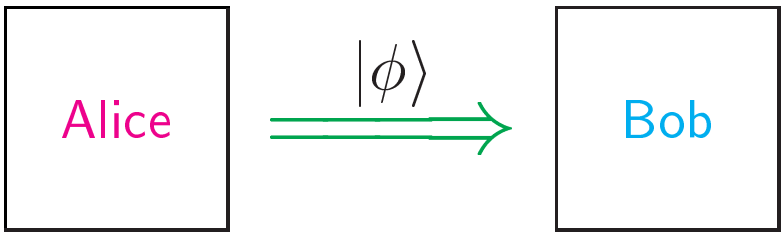
errors

e. g. photons
nuclear spins
trapped ions
optical trapped atoms
Josephson junction qubits
quantum dots

e. g. spontaneous emission
quantum jumps
interaction with environment
coupling to a bath

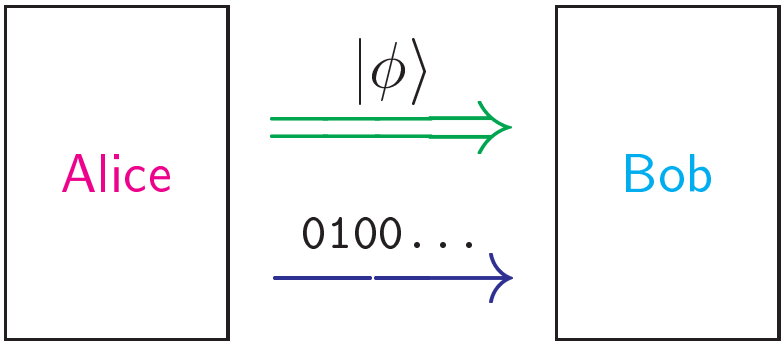
Quantum/Classical Channels

Quantum only:



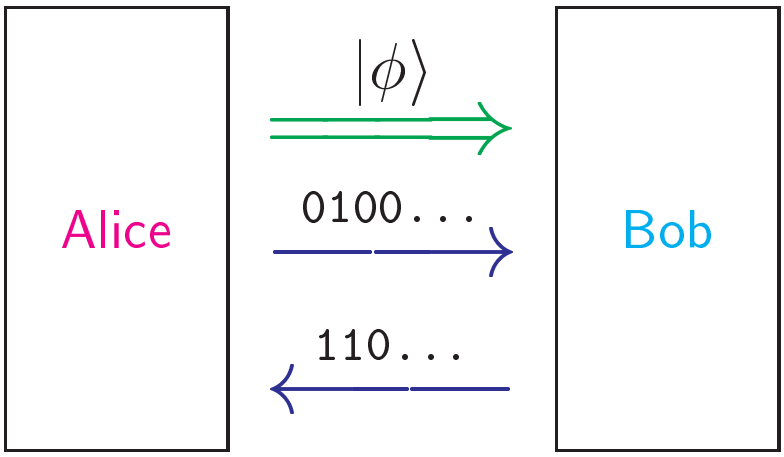
quantum error-correcting codes (QECC)

Quantum & classical forward-channel:



teleportation

Quantum & classical duplex channel:



entanglement purification & repeaters

Quantum Information

Quantum–bit (qubit)

basis states:

$$\text{“0”} \mathrel{\hat{=}} |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \in \mathbb{C}^2, \quad \text{“1”} \mathrel{\hat{=}} |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \in \mathbb{C}^2$$

general state:

$$|q\rangle = \alpha|0\rangle + \beta|1\rangle \qquad \text{where } \alpha, \beta \in \mathbb{C},\ |\alpha|^2 + |\beta|^2 = 1$$

Quantum register

basis states:

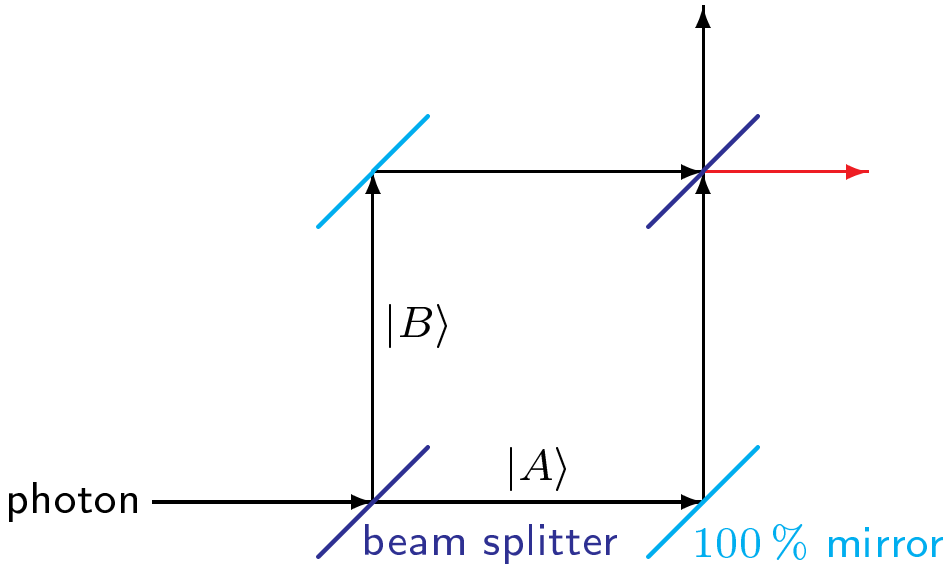
$$|b_1\rangle \otimes \ldots \otimes |b_n\rangle =: |b_1 \ldots b_n\rangle = |\boldsymbol{b}\rangle \qquad \text{where } b_i \in \{0,1\}$$

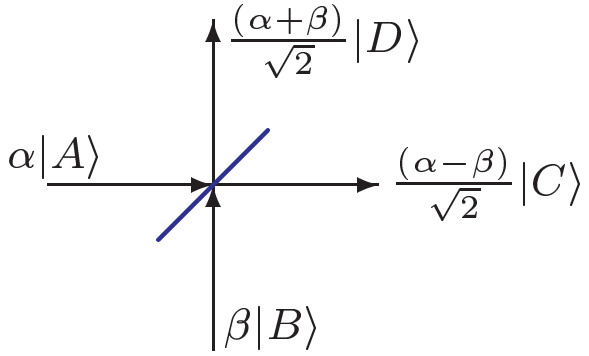
general state:

$$|\psi\rangle = \sum_{\boldsymbol{x} \in \{0,1\}^n} c_{\boldsymbol{x}} |x\rangle \qquad \text{where } \textstyle\sum_{\boldsymbol{x} \in \{0,1\}^n} |c_{\boldsymbol{x}}|^2 = 1$$

$\longrightarrow$ normalized vector in $(\mathbb{C}^2)^{\otimes n} \cong \mathbb{C}^{2^n}$

Interference or not?





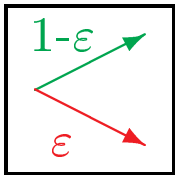
Interference: $(H \otimes \mathbb{1})\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|\,\epsilon\,\rangle\right) = |0\rangle|\,\epsilon\,\rangle$

No interference due to interaction:

$$(H \otimes \mathbb{1})\left(\frac{1}{\sqrt{2}}(|0\rangle|\epsilon_0\rangle + |1\rangle|\epsilon_1\rangle)\right) = \frac{1}{2}(|0\rangle + |1\rangle)|\epsilon_0\rangle + \frac{1}{2}(|0\rangle - |1\rangle)|\epsilon_1\rangle$$
$$= \frac{1}{2}|0\rangle(|\epsilon_0\rangle + |\epsilon_1\rangle) + \underbrace{\frac{1}{2}|1\rangle(|\epsilon_0\rangle - |\epsilon_1\rangle)}_{\text{error term}}$$

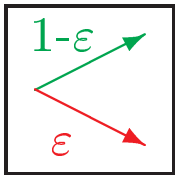
Quantum Channels: Examples

depolarizing channel

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \Longrightarrow$$ 

$$\Longrightarrow |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$
$$\Longrightarrow \text{random quantum state}$$

erasure channel

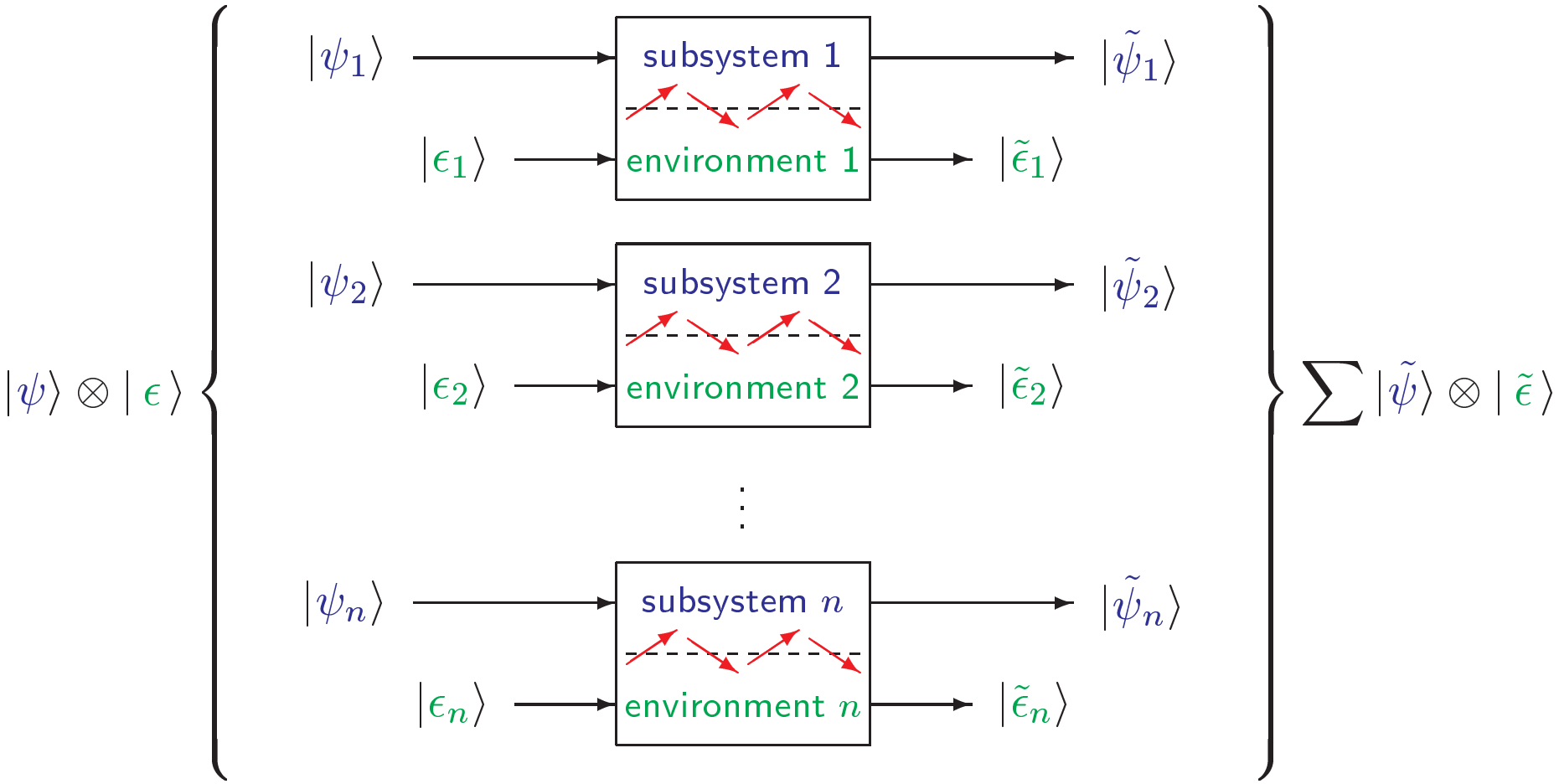
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \Longrightarrow$$ 

$$\Longrightarrow |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$
$$\Longrightarrow |E\rangle$$

where $\langle E|0\rangle = \langle E|1\rangle = 0$

Quantum Channel: Product Channel

Assumption: only *local interactions*, i. e., each subsystem interacts with a *separate* environment.



Repetition Code

classical

sender: repeated sending of the information,

e. g. $0 \mapsto 000,\ 1 \mapsto 111$

receiver: comparision of the received information and majority decision

quantum mechanical *Ansatz*:

sender: copying of the information,

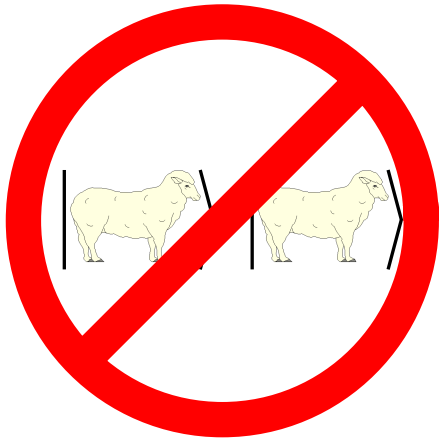
e. g. $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \mapsto |\psi\rangle|\psi\rangle|\psi\rangle$

receiver: comparision and majority decision

but: unknown quantum states can neither be copied nor

nor can they be disturbance-free compared

No-Cloning Theorem



- *unknown* quantum states cannot be copied:

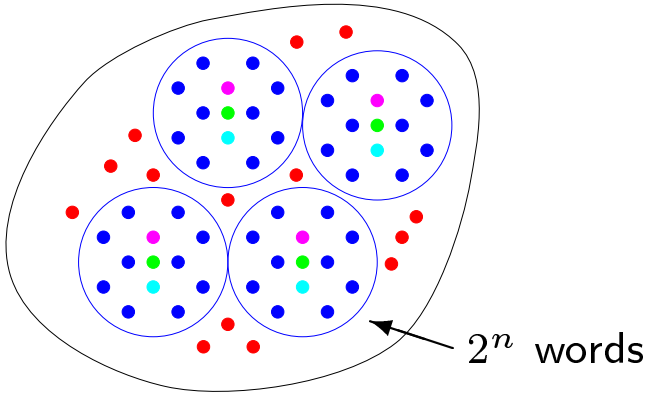
$$
\begin{aligned}
|0\rangle|\text{blank}\rangle &\mapsto |0\rangle|0\rangle \\
|1\rangle|\text{blank}\rangle &\mapsto |1\rangle|1\rangle \\
\hline
(\alpha|0\rangle+\beta|1\rangle)|\text{blank}\rangle &\mapsto \alpha|0\rangle|0\rangle+\beta|1\rangle|1\rangle \\
&\neq (\alpha|0\rangle+\beta|1\rangle)\otimes(\alpha|0\rangle+\beta|1\rangle) \\
&= \alpha^2|0\rangle|0\rangle+\beta^2|1\rangle|1\rangle \\
&\quad +\alpha\beta(|0\rangle|1\rangle+|1\rangle|0\rangle)
\end{aligned}
$$

- *unknown* quantum states can only be *statistically* compared

QECC: Basic Principle

Classical code

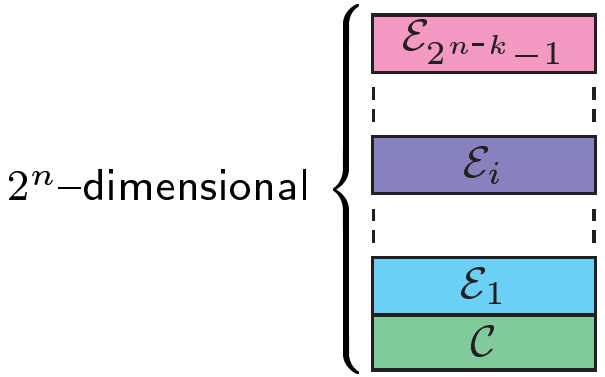
partitioning of the set of all words



- codewords
- errors of bounded weight
- other errors

Quantum code

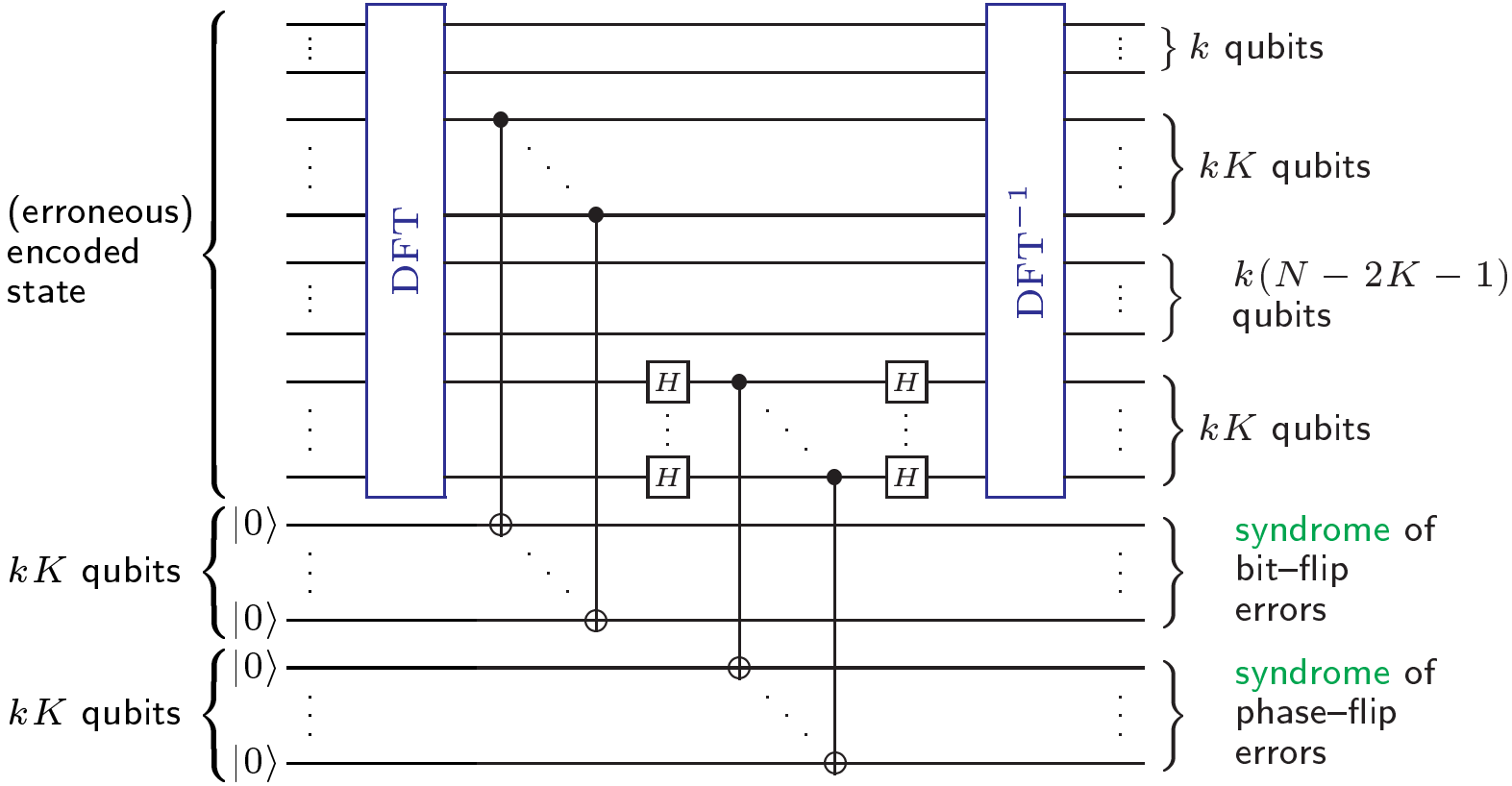
orthogonal decomposition of the vector space



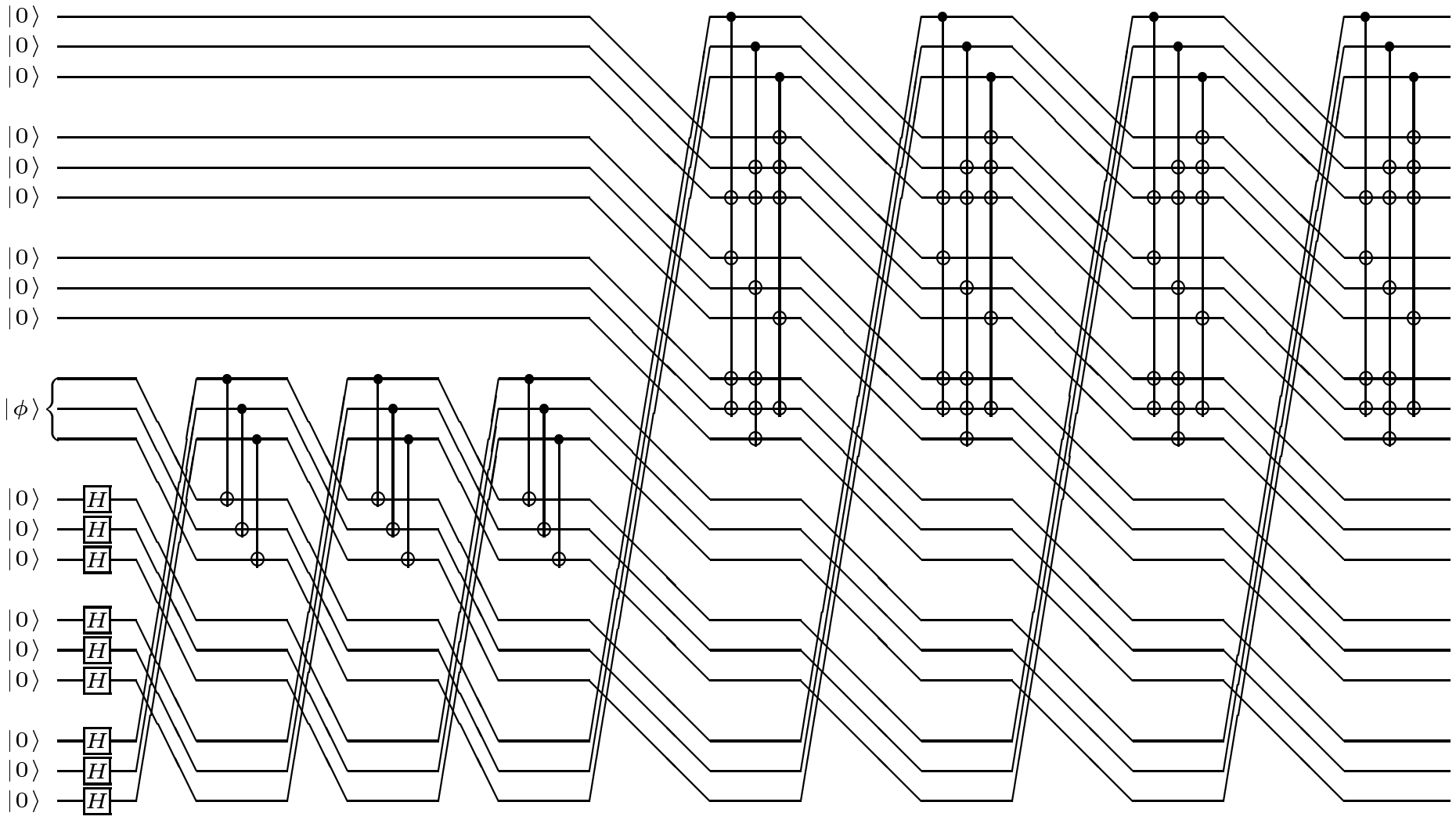
$$\mathbb{C}_2^{\otimes n} = \mathcal{H}_{\mathcal{C}} \oplus \mathcal{H}_{\mathcal{E}_1} \oplus \ldots \oplus \mathcal{H}_{\mathcal{E}_i} \oplus \ldots \oplus \mathcal{H}_{\mathcal{E}_{2^{n-k}-1}}$$

Quantum Reed-Solomon Codes

encoding/decoding using spectral techniques

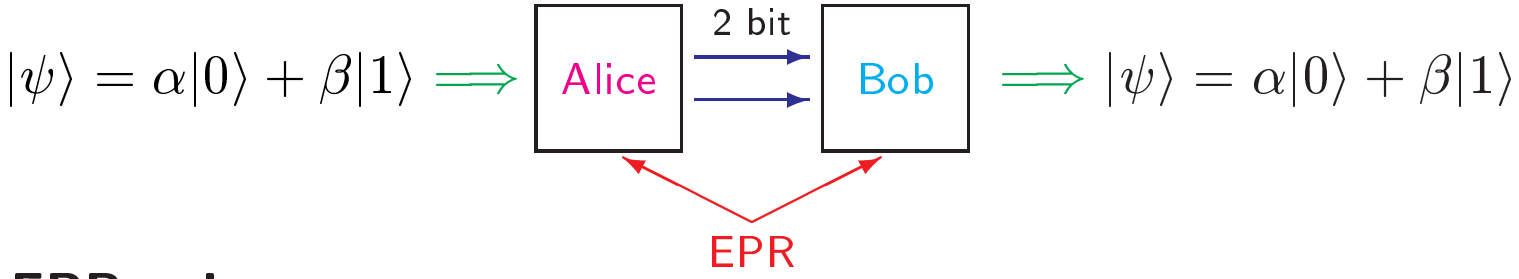


QLFSR-Encoder for a Cyclic QECC



Quantum Communication using Teleportation

transmitting quantum states using a classical channel:



needs EPR pairs:

$$\left|\Psi^-\right\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B\right)$$

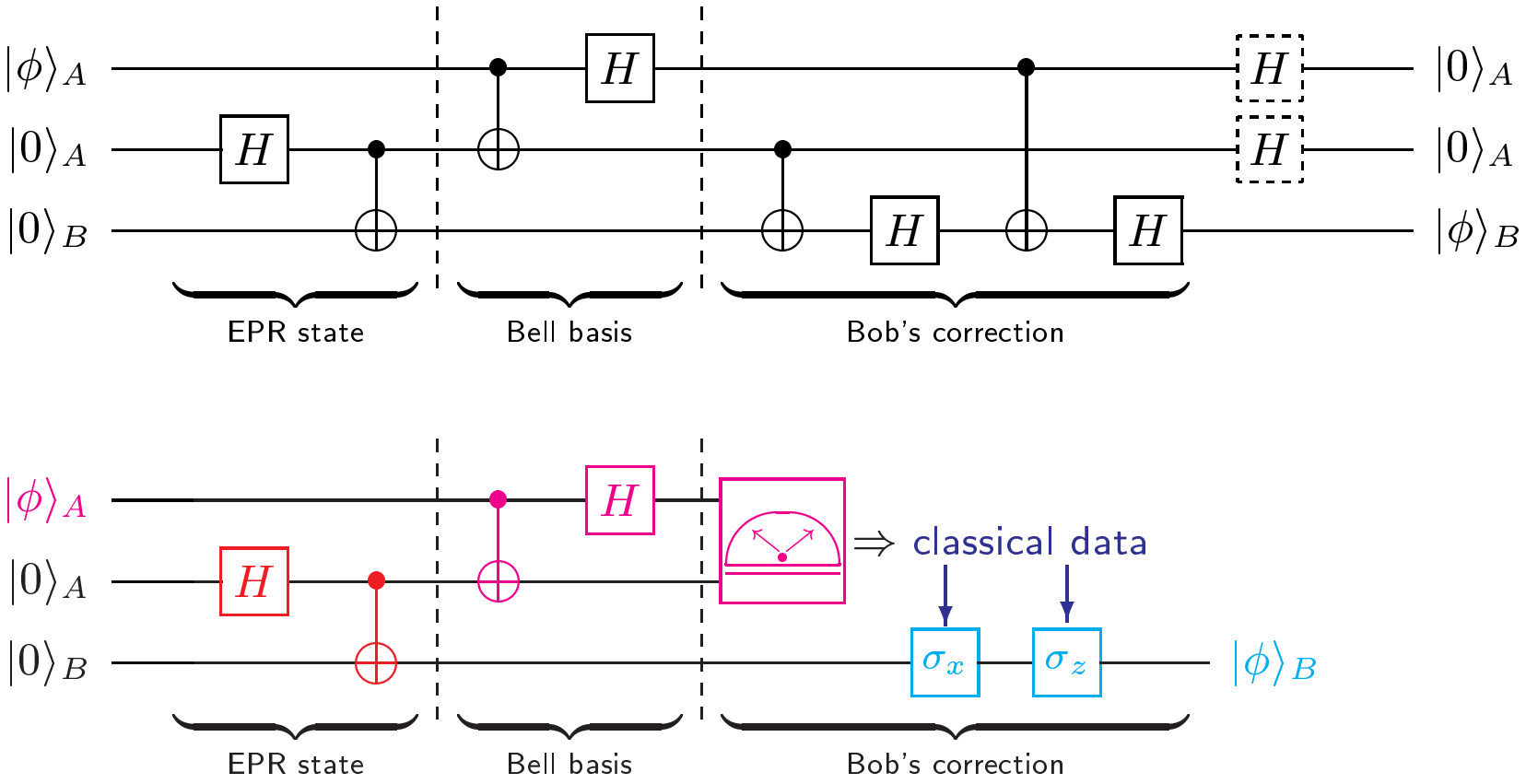
non-classical correlations (entanglement)

EPR pairs must be sent through a quantum channel:

but: – EPR pairs can be sent at any time (in the past)

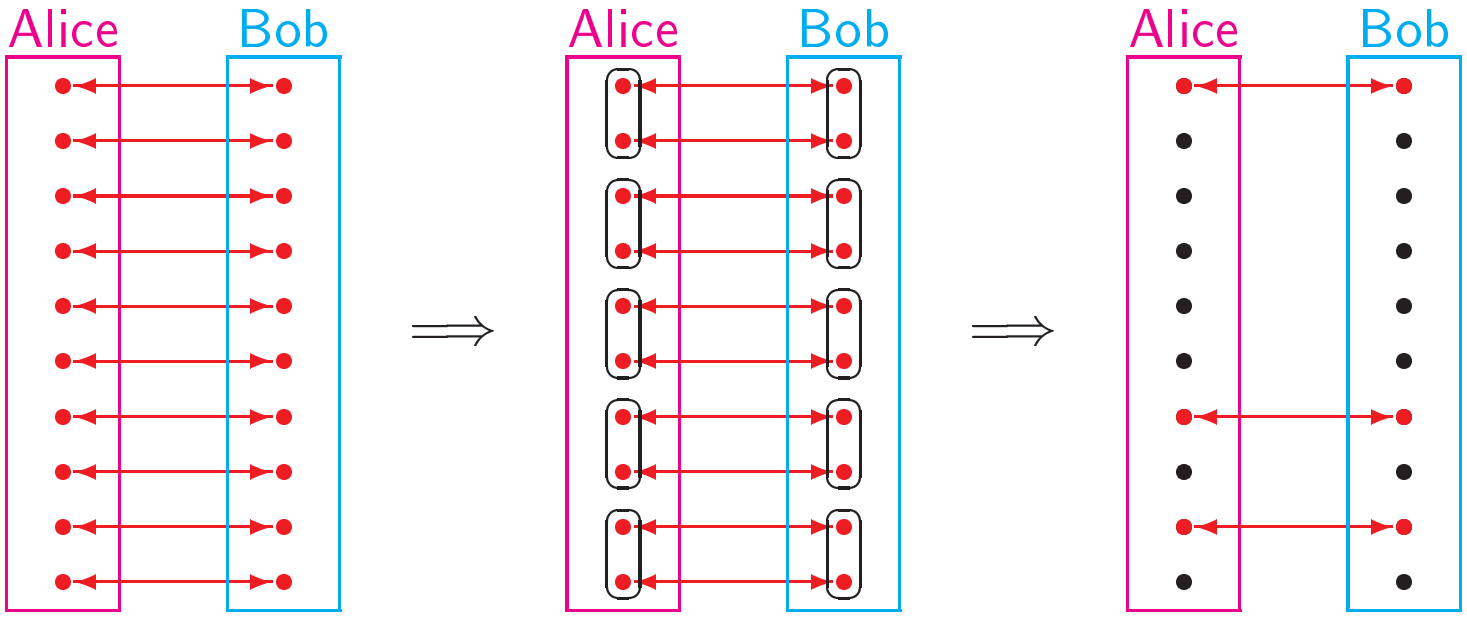
– EPR pairs may be purified

Quantum Circuit for Teleportation



Entanglement Purification

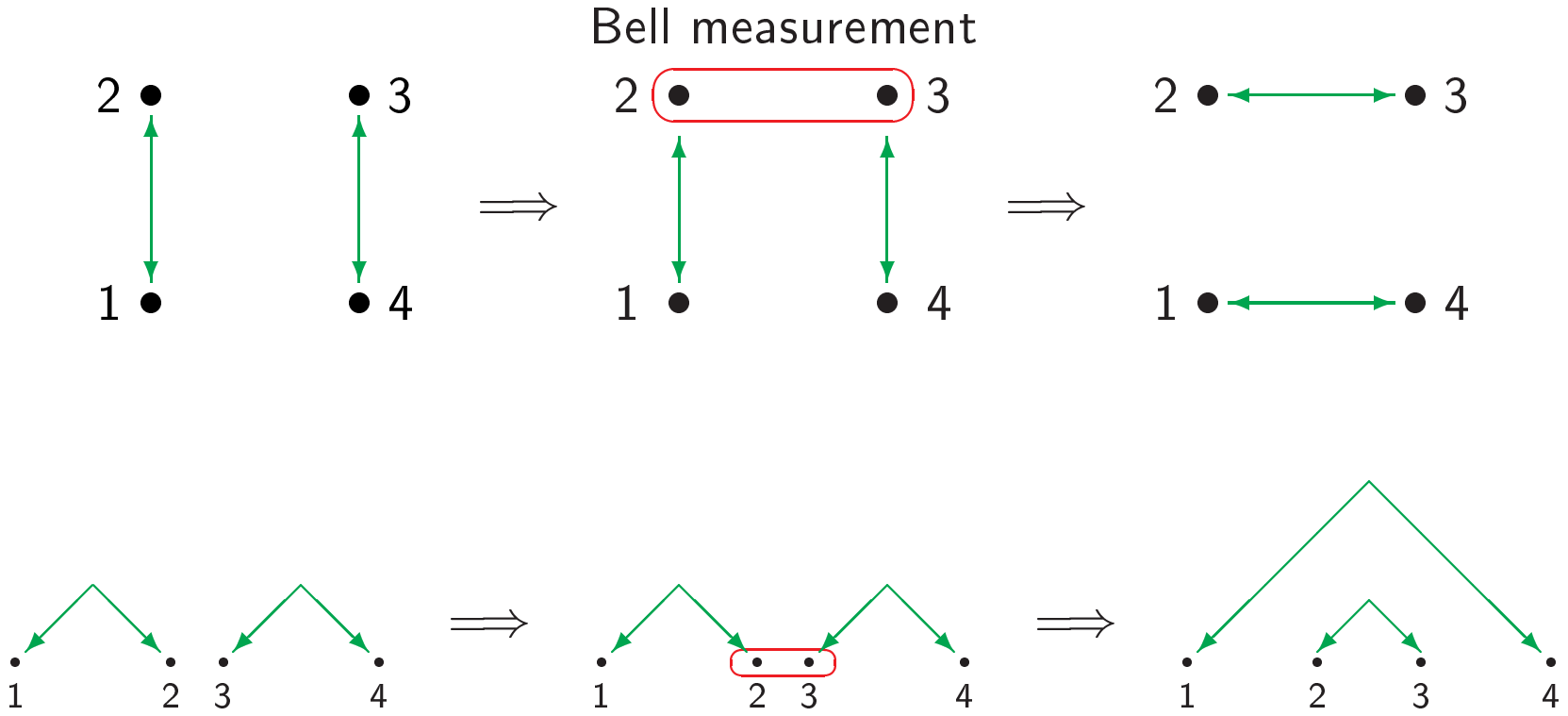
idea: reduce *many bad* EPR pairs to *some good* EPR pairs



needs classical duplex channel

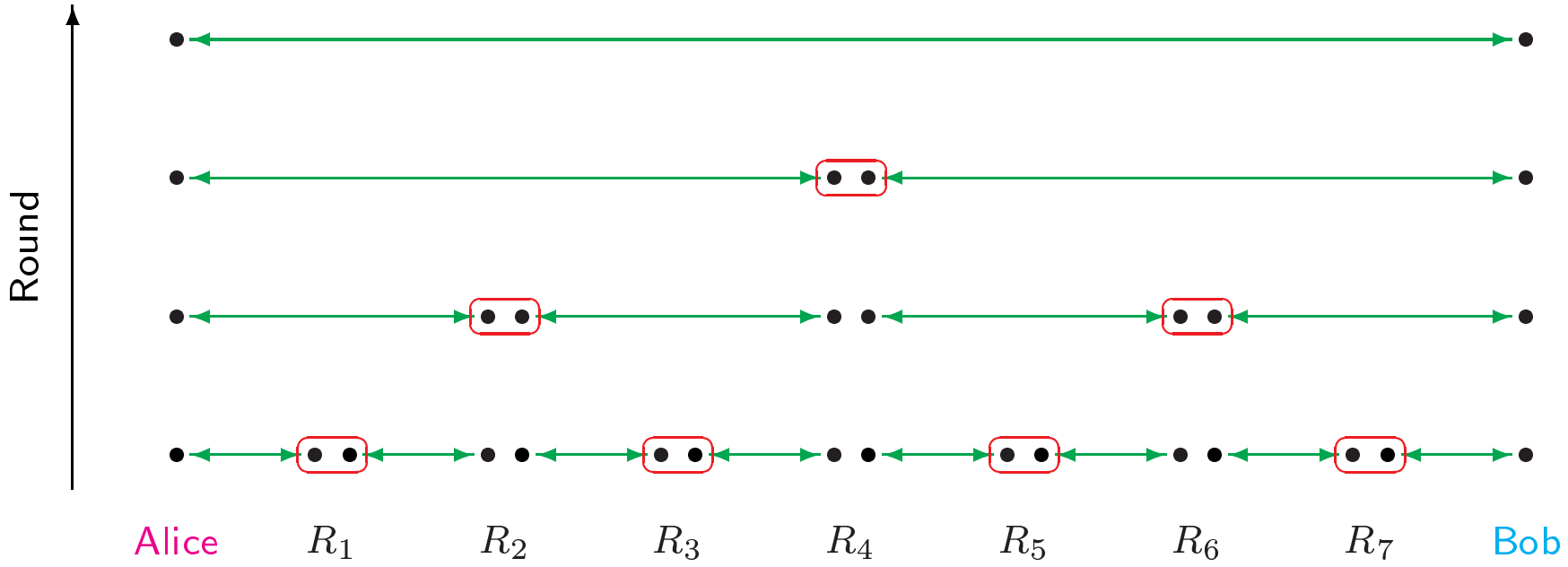
Entanglement Swapping

idea: indirect creation of entanglement



Quantum Repeaters

idea: create entanglement between distant particles



Related Topics

Error-free/decoherence-free/noiseless subspaces

– knowledge about system/environment interaction

– store information in "stable" subspaces

Retransmission in case of failure

– make "backup"

– send information

– test for transmission failure

– request retransmission in case of failure

Further information

– http://iaks-www.ira.uka.de/QIV

– http://xxx.lanl.gov/archive/quant-ph